MATH 141: Quiz 4

Name: Key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Given the equation

use chain rule $\sin(x^2y^3) = x^4 + y^2$ Use implicit differentiation.

Find $\frac{dy}{dx}$.

use product rule

$$\frac{d}{dx}\left[\sin(x^2y^3)\right] = \frac{d}{dx}\left[x^4\right] + \frac{d}{dx}\left[y^2\right]$$

left factor is x^2

right factor is y^3

$$\cos(x^2y^3)\cdot\frac{d}{dx}\left[x^2y^3\right] = 4x^3 + 2y\cdot\frac{dy}{dx}$$

$$\cos(x^2y^3)\left[y^3\cdot\frac{d}{dx}[x^2] + x^2\frac{d}{dx}[y^3]\right] = 4x^3 + 2y\frac{dy}{dx}$$

$$\underbrace{\cos(x^2y^3)}_{\text{factor}}\cdot\left(\overbrace{y^3\cdot 2x}^{\text{term}} + \overbrace{x^2\cdot 3y^2\cdot\frac{dy}{dx}}^{\text{term}}\right) = 4x^3 + 2y\frac{dy}{dx}$$

① Create global terms. No parentheses grouping terms.

$$2xy^3\cos(x^2y^3) + \underline{3x^2y^2\cos(x^2y^3)\cdot\frac{dy}{dx}} = 4x^3 + \underline{2y\frac{dy}{dx}}$$

② Put terms with $\frac{dy}{dx}$ on one side, terms without $\frac{dy}{dx}$ on the other.

$$3x^2y^2\cos(x^2y^3)\cdot\underline{\frac{dy}{dx}} - 2y\cdot\underline{\frac{dy}{dx}} = 4x^3 - 2xy^3\cos(x^2y^3)$$

③ Convert $\frac{dy}{dx}$ to a global factor.

$$\frac{dy}{dx}\cdot\underline{\left(3x^2y^2\cos(x^2y^3) - 2y\right)} = 4x^3 - 2xy^3\cos(x^2y^3)$$

④ Divide both sides by the factor next to $\frac{dy}{dx}$.

$$\boxed{\frac{dy}{dx} = \frac{4x^3 - 2xy^3\cos(x^2y^3)}{3x^2y^2\cos(x^2y^3) - 2y}}$$

2. If a circle is expanding over time, how fast is the area increasing when the radius is 4 cm and the radius is increasing at $\frac{1}{\pi}$ centimeters per second?

① A : area of circle

r : radius of circle

② $r = 4\,cm$, $\frac{dr}{dt} = \frac{1}{\pi}\ cm/s$

$\boxed{\text{want}}$: $\frac{dA}{dt}$

③ $A = \pi r^2$

④ $\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$

$$\frac{dA}{dt} = \pi \frac{d}{dt}[r^2]$$

$$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$$

⑤ N/A

⑥ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\cancel{\pi} \cdot 4\,cm \cdot \frac{1}{\cancel{\pi}}\ cm/s$

$$= \boxed{8\ cm^2/s}$$